Multibarrier tunneling

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We study the tunneling through an arbitrary number of finite rectangular opaque barriers and generalize earlier results by showing that the total tunneling phase time depends neither on the barrier thickness nor on the interbarrier separation. We also predict peculiar features of the system considered, namely the independence of the transit time (for nonresonant tunneling) and the resonant frequency on the number of barriers crossed, which can be directly tested in photonic experiments. A thorough analysis of the role played by interbarrier multiple reflections and a physical interpretation of the results obtained is reported, showing that multibarrier tunneling is a highly nonlocal phenomenon.

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I. INTRODUCTION

A renewed interest in a typical quantum phenomenon such as the tunnel effect has been recently achieved due to a long series of experiments aimed to measure the tunneling transit time (for reviews see, for instance, Ref. [1]). While such experiments involving electrons are usually difficult to realize (mainly due to the smallness of the electron de Broglie wavelength at usual temperatures) and even of uncertain interpretation, the observations on photonic tunneling [2–6] have by now provided clear data on this subject. Despite the different phenomena studied in several experiments (undersized waveguides, photonic band gap, total internal reflection) and the different frequency ranges for the light used (from the optical to the microwave region), all such experiments have shown that, in the limit of opaque barriers, the transit time to travel across a barrier of width a is usually *shorter* than the corresponding one required for real (not evanescent) propagation through the same region of width a. This result can be interpreted [1] in terms of a superluminal group velocity $v_{\mathrm{gr}}>c$ which, however, does not violate Einstein causality, since the signal velocity relevant for that [7] is never measured. Nevertheless, we prefer to look at the experimental result as an observation of the simple Hartman effect [8]: for opaque barriers the tunneling phase time is independent of the barrier width. Although several definitions of the tunneling time (also related to the different experimental setups used) exist [1] and a general consensus on this is still lacking, it seems that all the experimental results can be successfully interpreted in terms of phase time [9].

Further light has been put on the problem by recent experiments involving double-barrier penetration [10]. In fact, while the above effect has been confirmed in such a system too (far from the resonances of the structure), observations show that the transit time is also independent of the separation distance between the barriers (supposed to be thick). This peculiar phenomenon has been studied theoretically in Ref. [11], where the authors have provided a straightforward generalization of the Hartman effect for double-barrier tunneling.

Convincing qualitative explanations of these two findings (namely, that the tunneling phase time is independent of the barrier thickness as well as of the interbarrier separation for opaque barriers) have been reported. When considering a given wave packet entering a potential barrier region, a reshaping phenomenon occurs in which the traveling edge of the pulse is preferentially attenuated with respect to the leading one, thus simulating a group velocity greater than c [1,3]. In practice, the Hartman effect in the tunneling through a thick barrier is explained from the fact that inside the barrier no phase accumulates, and the entire phase shift comes only from the boundaries, thus being substantially independent of the thickness [12]. Furthermore, when two barriers are present, the transit time independence on the barrier separation can, instead, be understood in terms of an effective acceleration of the forward travelling waves in the interbarrier region, which arises from a destructive interference between the two barriers [11].

Further noticeable results have been recently achieved in Ref. [12], where it has been shown that a wave packet travels *in zero time* a region with N arbitrary δ-function barriers.

In this paper, we extend all these findings by considering the case of N successive opaque barriers with finite widths and heights. While we confirm all previous results, we generalize them by showing that some peculiar tunneling properties are independent of the number of the barriers crossed (Sec. II). Furthermore, in order to establish a quantitative interpretation of the involved phenomena, in Sec. III we study the role of multiple reflections in double-barrier tunneling and show how strongly the total tunneling phase time depends on nonlocal effects. Finally, in Sec. IV we discuss the results obtained and give our conclusions.

In view of the formal analogy [13] between the Schrödinger equation and the electromagnetic Helmholtz equation, our study applies to matter particle tunneling as well as to evanescent propagation of photonic wave packets. This is a straightforward consequence of the fact that in both cases the starting point is basically the same [in our case it is Eq. (2)] [9], on interchanging the roles of angular frequency ω and

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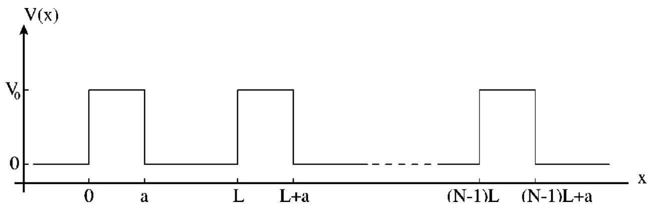

FIG. 1. Potential barrier $V(x)$ with N equally spaced rectangular bumps of given height and width.

wave vector k into the corresponding ones of energy E and momentum p through the Planck–de Broglie relations. Thus, throughout this paper, we indifferently use particle or wave terminology unless the meaning of what we are doing becomes unclear.

II. TUNNELING THROUGH *N* SUCCESSIVE BARRIERS

Let us consider a wave packet moving along the x axis and entering at $x=0$ a region with a potential barrier $V(x)$ as depicted in Fig. 1:

$$V(x)=\begin{cases} V_0, & (i-1)L\leqslant x\leqslant (i-1)L+a \\ 0, & \text{otherwise,}\end{cases} \tag{1}$$

for $i=1,2,\ldots,N$. For the sake of simplicity, we choose the height V_0 of the potential barriers, as well as the width a of each barrier, to be the same for all N rectangular barriers. We further assume equally spaced barriers, $L-a$ being the inter-barrier distance.

The propagation of the wave packet through the barriers is described by a scalar field ψ representing the Schrödinger wave function in the particle case or some scalar component of the electric or magnetic field in the photonic case. This is the solution of the Schrödinger equation or the Helmholtz equation with potential or refractive index in Eq. (1) and, in both cases, it takes the following form:

$$\psi(x)=\begin{cases} \psi_{2i}(x), & (i-1)L\leqslant x\leqslant (i-1)L+a \\ & (i=1,2,\ldots,N) \\ \psi_{2i+1}(x), & \text{otherwise} \\ & (i=0,1,2,\ldots,N),\end{cases} \tag{2}$$

with:

$$\psi_1(x)=e^{ikx}+Re^{-ikx}, \tag{3}$$

$$\psi_{2i}(x)=A_{2i}e^{\chi[x-(i-1)L]}+B_{2i}e^{-\chi[x-(i-1)L]}(i=1,2,\ldots,N), \tag{4}$$

$$\psi_{2i+1}(x)=A_{2i+1}e^{ik[x-(i-1)L]}+B_{2i+1}e^{-ik[x-(i-1)L]} \times (i=1,2,\ldots,N-1), \tag{5}$$

$$\psi_{2N+1}(x)=Te^{ik[x-(N-1)L]}. \tag{6}$$

Obviously, the physical field is represented by a wave packet with a given spectrum in ω:

$$\Psi(x,t)=\int d\omega\, \eta(\omega)\psi(x)e^{-i\omega t},$$

where $\eta(\omega)$ is the envelope function. Keeping this in mind, for the sake of simplicity we deal with only stationary solutions as in Eq. (2).

An alternative parametrization for the wave function (which is especially useful for large N) is that of writing $\psi(x)$ in Eq. (2) in terms of the periodic and evanescent Bloch wave functions of the corresponding periodic barrier potential. This approach also allows to obtain closed expressions for the reflection and transmission coefficients for an arbitrary number of barriers [14]. However, we prefer to start with Eq. (2) which is a direct generalization of the corresponding expression usually considered in one-barrier systems, in view of our discussion on analogies and departures between the N-barrier and single-barrier cases. As noted in Ref. [9], the explicit dependence on the frequency of the (real) wave vector k in the barrier-free regions and imaginary wave vector $i\chi$ in the barrier ones enters only the final expression for the phase time. As long as possible we do not use a particular dispersion relation in order to draw general features which are common to the particle and to the wave case.

The $4N$ unknown coefficients R,T,A_i,B_i are obtained from the $4N$ matching conditions for the function ψ and its derivative ψ' at the discontinuity points $x+(i-1)L$, $x=(i-1)L+a$ of the potential. Note that the quantities R and T have the meaning of (total) reflection and transmission coefficient from the N-barrier system, respectively, and satisfy the unitarity condition $|R|^2+|T|^2=1$.

We have produced a MATHEMATICA symbolic code in order to obtain explicit analytic expressions for all the coefficients appearing in Eqs. (3)–(6). However, we here report only the interesting result obtained for the transmission coefficient $T(N)$ for an N-barrier system in the opaque barrier approximation[1] $\chi a\gg 1$. In this limit the quantity $T(N)$ can be factorized in the following way:

$$T(N)e^{ika}=\mathcal{C}_0\mathcal{E}(N)\mathcal{F}(N), \tag{7}$$

$$\mathcal{C}_0=\frac{4i\chi k}{(k+i\chi)^2},$$

$$\mathcal{E}(N)=[e^{-\chi a}]^N,$$

$$\mathcal{F}(N)=\left[\frac{2\chi k}{2\chi k\cos k(L-a)-(k^2-\chi^2)\sin k(L-a)}\right]^{N-1}.$$

Note that only the real terms $\mathcal{E}$ and $\mathcal{F}$ depend on a,L,N, while the complex factor $\mathcal{C}_0$ does not. As a consequence, since the tunneling phase time τ is defined as

[1]For general expressions obtained using the parametrization in terms of Bloch wave functions, see Ref. [14].

$$\tau = \frac{d\phi}{d\omega} \tag{8}$$

and the quantity

$$\phi \equiv \arg\{T(N)e^{ika}\} = \arg\left\{\frac{4i\chi k}{(k+i\chi)^2}\right\}$$
$$= \arctan\frac{k^2-\chi^2}{2\chi k} \tag{9}$$

is independent of a,L,N, we arrive at the general conclusion that *the tunneling phase time for a system of N opaque barriers depends neither on the barrier width and interbarrier distance nor on the number of the barriers*.

Let us now discuss the effects of the real terms in Eq. (7) on the tunneling probability $P_T(N)=|T(N)|^2$:

$$P_T(N) = \left[\frac{4\chi k}{k^2+\chi^2}\right]^2 [e^{-\chi a}]^{2N} \times \left[\frac{2\chi k}{2\chi k\cos k(L-a)-(k^2-\chi^2)\sin k(L-a)}\right]^{2(N-1)}. \tag{10}$$

We easily recognize that the last factor in Eq. (10), coming from the term $\mathcal{F}(N)$ is responsible for the resonance structure of the transmission probability. The factor $\mathcal{F}(N)$, is, of course, absent in the case of only one barrier, i.e., $N=1$ or $N\neq 1$ but $L=a$. However, no resonance can occur even in the particular case in which the interbarrier distance is tuned in a way that $L-a=\nu\pi/k(\nu=0,1,2,\dots)$. In this case, waves moving forward and backward in the interbarrier regions interfere between them such that no resonance takes place. The resonance condition for the tunneling probability is, from Eq. (10), the following:

$$\tan k(L-a) = \frac{2\chi k}{k^2-\chi^2}. \tag{11}$$

It is worthwhile to observe that Eq. (11) does not depend on N, so that *the resonant frequency is the same irrespective of the number of barriers to be crossed*. Note, however, that the $N-1$ coincident resonances of Eq. (10) of the ideal case studied here split into $N-1$ closely spaced (but different) resonances in real physical systems. For example, in crystals, in the limit of infinite N, these merge into the band structure of the periodic barrier potential.

Finally, we point out an intriguing consequence of the resonance condition. Let us write Eq. (11) as follows:

$$\tan\phi\tan k(L-a) = 1, \tag{12}$$

where ϕ is given in Eq. (9), and take the derivative of Eq. (12) with respect to the angular frequency ω. By using Eq. (8), we easily recognize that $\tau+\tau_0=0$, where τ_0 is the (phase) time for traveling the interbarrier distance $L-a$ in vacuum. Keeping in mind that the total tunneling time has the same value of the tunneling time for crossing only one barrier (see above), we see that, when resonant tunneling takes place, *the total time required to cover the distance L* (one barrier length a plus one interbarrier distance $L-a$) *is zero*. This, however, is only a mathematical result since, in the actual situation, the physical phase time is the sum of the off-resonance time considered above (for which the property just outlined holds) and the time required to cross the resonance, which is typically much larger than the off-resonance time, being proportional to the absolute thickness of the potential structure. In the following, we only consider the interesting case of nonresonant tunneling.

III. MULTIPLE REFLECTIONS AND NONLOCALITY

In order to have a physical interpretation of the results obtained previously, we now consider the effect of single barriers on the propagation of the wave packet through the entire N-barrier system, by invoking the superposition principle. For xsthe sake of simplicity, we will study the case of a system of two opaque barriers.

A. Partial coefficients

For $N=2$, in the barrier-free regions, Eqs. (3)–(6) reduce to the following:

$$\psi_1(x) = e^{ikx}+Re^{-ikx},$$
$$\psi_3(x) = A_3e^{ikx}+B_3e^{-ikx}, \tag{13}$$
$$\psi_5(x) = Te^{ik(x-L)},$$

where the explicit expressions for the coefficients are reported in the Appendix. Let us now denote with R_1,T_1 and R_2,T_2 the (partial) reflection and transmission coefficients of the first and second barrier, respectively. In the region with $x<0$ the reflected wave is described by the term

$$Re^{-ikx} = R_1e^{-ikx}+B_3T_1e^{-ikx}, \tag{14}$$

while for $x>L+a$ the transmitted one is described by

$$Te^{ik(x-L)} = A_3T_2e^{ik(x-L)}. \tag{15}$$

By taking into account multiple reflections from the two barriers in the region with $a<x<L$, we see that the forward traveling wave is described by the term

$$A_3e^{ikx} = T_1[1+R_1R_2+(R_1R_2)^2+\cdots]e^{ikx}, \tag{16}$$

while the backward one is described by

$$B_3e^{-ikx} = A_3R_2e^{-ik(x-L)}. \tag{17}$$

Then, by introducing the quantity

$$S = \sum_{l=0}^{\infty}(R_1R_2)^l = \frac{1}{1-R_1R_2}, \tag{18}$$

which accounts for multiple reflections, from Eqs. (14)–(17) we obtain

$$R = R_1 + B_3 T_1,$$
$$T = A_3 T_2, \tag{19}$$
$$A_3 = T_1 S,$$
$$B_3 = A_3 R_2 e^{ikL}.$$

By solving these equations with respect to the partial reflection and transmission coefficients, we get

$$R_1 = \frac{R - A_3 B_3}{1 - B_3^2},$$
$$T_1 = \frac{A_3 - B_3 R}{1 - B_3^2}, \tag{20}$$
$$R_2 = \frac{B_3}{A_3} e^{-ikL},$$
$$T_2 = \frac{T}{A_3}.$$

In the opaque barrier limit $\chi a \gg 1$, for the second barrier we obtain

$$R_2 = R_{\mathrm{OB}} e^{ikL}, \tag{21}$$
$$T_2 = T_{\mathrm{OB}} e^{ikL},$$

while for the first barrier:

$$R_1 = R_{\mathrm{OB}} + R_{\mathrm{Q}} + R_{\mathrm{R}}, \tag{22}$$
$$T_1 = T_{\mathrm{OB}} + T_{\mathrm{Q}} + T_{\mathrm{R}},$$

where

$$R_{\mathrm{OB}} = \frac{k - i\chi}{k + i\chi}\left[1 - \frac{4i\chi k}{(k+i\chi)^2} e^{-2\chi a}\right],$$
$$T_{\mathrm{OB}} = \frac{4i\chi k}{(k+i\chi)^2} e^{-ika} e^{-\chi a} \tag{23}$$

are the reflection and transmission coefficients corresponding to a one-barrier system $(N=1)$ and

$$R_{\mathrm{Q}} = -\left(\frac{k - i\chi}{k + i\chi}\right)^3 \mathcal{F}^2 e^{2ik(L-a)} e^{-2\chi a},$$
$$R_{\mathrm{R}} = \left(\frac{k - i\chi}{k + i\chi}\right)^3 \mathcal{F}^2 e^{ikL} e^{-2\chi a},$$
$$T_{\mathrm{Q}} = \left(\frac{k - i\chi}{k + i\chi}\right)^2 \mathcal{F} e^{2ik(L-a)} e^{-ikL} e^{-\chi a}, \tag{24}$$
$$T_{\mathrm{R}} = -\left(\frac{k - i\chi}{k + i\chi}\right)^2 \mathcal{F} e^{-\chi a} \quad .$$

For future reference, we also consider the partial coefficients $R_1^0, T_1^0, R_2^0, T_2^0$ in the approximation of no multiple reflections, as considered in Ref. [11]. These are obtained from Eqs. (19) by setting $S=1$. We have

$$R_1 = R_{\mathrm{OB}} + R_{\mathrm{Q}}, \tag{25}$$
$$T_1 = T_{\mathrm{OB}} + T_{\mathrm{Q}} \quad ,$$

while R_2^0, T_2^0 are the same as in Eqs. (21).

Note, however, that in Ref. [11] the authors have considered the case of no multiple reflections and, moreover, they also neglected the second term $B_3 T_1$ in the first equation in Eq. (19) corresponding to backward waves in the $x<0$ region transmitted from the first barrier, reflected from the second one and again transmitted from the first barrier. In this approximation, the quantity R_1^0 in Eq. (25) should be replaced by the following one:

$$\widetilde{R}_1^0 = R_{\mathrm{OB}} + \frac{k - i\chi}{k + i\chi} \frac{4i\chi k}{(k+i\chi)^2} \mathcal{F} e^{ik(L-a)} e^{-2\chi a}.$$

While the parametrization of the wave function considered in Ref. [11] is, of course, permitted and leads to correct results, nevertheless, the partial coefficients they obtained have no direct physical meaning, as we will show below.

B. Unitarity conditions

The interpretation of the quantities R_1, T_1 and R_2, T_2 as reflection and transmission coefficients of the first and second barrier is derived from the unitarity conditions satisfied by these coefficients. In fact, since $|R|^2 + |T|^2 = 1$ and $|R_{\mathrm{OB}}|^2 + |T_{\mathrm{OB}}|^2 = 1$, we find that

$$|R_1|^2 + |T_1|^2 = 1, \tag{26}$$
$$|R_2|^2 + |T_2|^2 = 1.$$

It is easily recognizable as well that, assuming no multiple reflection, the total probability for scattering from the first barrier is *lower* than 1:

$$|R_1^0|^2 + |T_1^0|^2 = 1 - \mathcal{F}^2 e^{-2\chi a}, \tag{27}$$

this revealing that something has been forgotten. Obviously, multiple reflections are the missing terms and it is worth to observe that the probability for this phenomenon to occur, which from Eq. (27) we deduce to be $\mathcal{F}^2 e^{-2\chi a}$, is given by

$$P_{\mathrm{R}} \equiv |R_{\mathrm{R}}|^2 + |T_{\mathrm{R}}|^2 = \mathcal{F}^2 e^{-2\chi a}. \tag{28}$$

Thus the quantities R_{R} and T_{R}, which must be added to the no multiple reflection coefficients R_1^0 and T_1^0, in order to obtain the complete ones R_1 and T_1 respectively, can be interpreted as the terms describing the phenomenon of multiple reflections between the first and second barrier.

Incidentally, by using the parametrization of Ref. [11], we obtain an unphysical scattering probability *greater* than 1,

$$|\tilde{R}_1^0|^2+|T_1^0|^2=1+\mathcal{F}^2 e^{-2\chi a},$$

which makes it impossible to give a direct physical meaning to $\tilde{R}_1^0$, T_1^0.

The meaning of the picture just outlined is then quite trivial. R_2 and T_2 corresponding to the second barrier are simply given by the one-barrier coefficients $R_{\rm OB}, T_{\rm OB}$ times a phase factor which takes into account the fact that the second barrier starts at $x=L$, while the reference point in our discussion is at $x=0$. Instead, R_1 and T_1 related to the first barrier are given by the sum of two terms: the first one corresponds to the no multiple reflection coefficients, while the second one describes the phenomenon of multiple reflections. However, it is remarkable that *no multiple reflection coefficients* R_1^0 *and* T_1^0 *in Eqs. (25) do not coincide with the one-barrier coefficients* $R_{\rm OB}$ *and* $T_{\rm OB}$. This is an obvious consequence of the fact that the scattering probability from the first barrier, *neglecting* multiple reflections, cannot be equal to unity and the extra terms $R_{\rm Q}$ and $T_{\rm Q}$ in Eqs. (25) must be present in order to achieve the probability constraint in Eq. (27). On the other hand, the scattering probability, *including* multiple reflections, must be equal to 1 [according to Eq. (26)], so that we can deduce that the quantities $R_{\rm Q}$ and $T_{\rm Q}$ are related to the multiple reflection coefficients $R_{\rm R}$ and $T_{\rm R}$. It is very easy to obtain from Eqs. (24) that $R_{\rm Q}$ and $T_{\rm Q}$ differ from $R_{\rm R}$ and $T_{\rm R}$ just by a phase factor (depending on L and a):

$$\frac{R_{\rm Q}}{R_{\rm R}}=\frac{T_{\rm Q}}{T_{\rm R}}=-e^{ik(L-2a)}. \tag{29}$$

Then, multibarrier tunneling is a highly nonlocal phenomenon driven by multiple reflections, whose influence on the determination of the reflection and transmission coefficients is (indirectly) present even in the case in which they are neglected.

C. Tunneling phase time

Let us now consider the tunneling phase time τ in Eq. (8) corresponding to the double-barrier crossing considered here and introduce the quantities

$$\phi_1=\arg\{T_1 e^{ika}\},$$

$$\phi_2=\arg\{T_2 e^{ika}\}, \tag{30}$$

$$\phi_{\rm S}=\arg\{S e^{ik(L-a)}\},$$

whose derivatives with respect to frequency give the phase times for the first-barrier crossing, the second-barrier crossing, and the time associated to multiple reflections, respectively. Since $T=T_1T_2S$ from Eqs. (19), the total tunneling phase is given by

$$\phi=\phi_1+(\phi_2-kL)+\phi_{\rm S}. \tag{31}$$

This relation leads to the obvious conclusion that the tunneling time τ is the sum of the partial times[2] $2\tau_1$ and τ_2 spent to travel across the first and second barrier, respectively, plus the time $\tau_{\rm S}$ required by multiple reflections in the interbarrier region of length $L-a$. However, it is interesting to evaluate the explicit expressions for these times and, from Eqs. (30) we get

$$\phi_1=\phi_0-\frac{kL}{2}+ka, \tag{32}$$

$$\phi_2-kL=\phi_0, \tag{33}$$

where $\phi_0=\arg\{T_{\rm OB}e^{ika}\}$ is the one-barrier tunneling phase time. For opaque barriers, the leading term in S is, from Eqs. (18), (21), and (22), the following:

$$S=\frac{(k+i\chi)^2}{4i\chi k}e^{-ikL/2}\frac{2\chi k}{2\chi k\cos kL/2-(k^2-\chi^2)\sin kL/2}$$

and thus

$$\phi_{\rm S}=-\phi_0+\frac{kL}{2}-ka. \tag{34}$$

While the time required to cross the second barrier equals exactly the one-barrier tunneling phase time [see Eq. (33)], from Eqs. (32) and (34) we see that

$$\phi_1+\phi_{\rm S}=0, \tag{35}$$

that is, *the time spent in traveling from the starting edge of the first barrier to the starting edge of the second one is zero*. Something similar to this statement has already been suggested in literature (see, for instance, Ref. [11]), but now we have a quantitative proof for that. Moreover, we can also deduce that, due to multiple reflections, the time to cross the first barrier is usually *different* from the one-barrier tunneling phase time since

$$\phi_0-\phi_1=\frac{\phi_{\rm Q}-\phi_{\rm R}}{2}, \tag{36}$$

where

$$\phi_{\rm Q}=\arg\{T_{\rm Q}\}=2\phi_0+kL-2ka,$$

$$\phi_{\rm R}=\arg\{T_{\rm R}\}=2\phi_0$$

are the phases corresponding to the terms $T_{\rm Q}$ and $T_{\rm R}$, the equality holding true only in the case in which the interbarrier distance coincides with the barrier width, i.e., $L=2a$.

[2]Note that the time τ_2 corresponds to the phase ϕ_2-kl, since the traveling along the distance L is already taken into account in $\phi_1+\phi_{\rm S}$ or, in other words, in the expression for the coefficient T_2 in Eq. (21) we have already considered the shift from $x=0$ to $x=L$.

IV. CONCLUSIONS

In this paper we have studied the tunneling of a particle or a photonic wave packet through an arbitrary number N of finite rectangular opaque barriers and obtained an analytic expression for the total transmission coefficient Eq. (7). From this we have confirmed and generalized to the present case what was found earlier for a system of one [8] or two [11] barriers: the (total) tunneling phase time is independent of both the barrier width and interbarrier distance. The same result applies to the reflection time for the model studied in the present paper. In fact the potential barrier considered here is symmetric, so that the reflection phase and the transmission phase only differ by the fixed angle $\pi/2$. As a result, the phase times for reflection and transmission are equal. These features have been observed experimentally for single- [2–6] and double-barrier [10] tunneling using photonic setups. Amazingly enough, we have further found that, although the tunneling probability decreases exponentially with increasing barrier thickness and the number of barriers (in the opaque barrier limit), the tunneling time does not depend even on the number of barriers crossed, i.e., it is the same for one, two or more barriers. Moreover, when considering resonant tunneling, we have also shown that the position in frequency (or energy) of the resonance of the structure is independent of the number of barriers as well. These two predictions can be experimentally tested using, again, photonic devices.

In order to obtain a physical picture of what happens in the system considered and, especially, of the peculiar features outlined above, we have studied the role of multiple reflections between the barriers on the tunneling and found this to be a highly nonlocal phenomenon. In fact, as shown in Sec. III, even in the case of increasingly large separation between the barriers, the effect of multiple reflections cannot be avoided at all. In particular, multiple reflections play a crucial role in the understanding of the intriguing results on the (total) tunneling time quoted above. Though in Sec. III we have dealt with a two-barrier system, the main results achieved can be easily generalized to multibarrier tunneling as follows. For N barriers the partial reflection and transmission coefficients corresponding to the first $N-1$ barriers are clearly influenced by multiple reflections occurring in the interbarrier regions, while those associated to the last barrier are not and coincide with one-barrier coefficients up to a phase factor. In particular, as shown in Sec. III C, the tunneling phase time for crossing only the last barrier equals that for a single-barrier structure. Since the total tunneling time for crossing all the barriers coincides as well with the one-barrier time (see Sec. II), we immediately deduce that the time for traveling from the starting edge of the first barrier to the starting edge of the last one is zero. Note that such a result can be achieved only if we take into account multiple reflections and, in any case, the partial times for crossing single barriers are usually different from the one-barrier tunneling time. However, we stress that such a "partial time" is not directly measured in physical experiments and, as a consequence, is not completely meaningful. Nevertheless, our discussion results to be useful in pointing out the relevant role of multiple reflections.

Finally, we point out that our findings also agree with the recent results reported in Ref. [12], according to which a wave packet travels in zero time a region with N δ-function barriers. In fact, as said above, the total tunneling time coincides with the transit time for the last barrier or one-barrier phase time. From Ref. [9] (see Eq. (13) of that paper) we then see that, for $\chi\rightarrow\infty$, this time tends to zero, thus recovering the result of Ref. [12]. It would then be nice, in the future, to make the connection between multiple reflections studied here and the tunneling interpretation in terms of superoscillations quoted in Ref. [12].

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APPENDIX: COEFFICIENTS FOR $N=2$

From the matching conditions for the wave function in Eq. (2), we obtain the following expressions for the coefficients describing the propagation through two successive opaque barriers:

$$R\simeq\frac{k-i\chi}{k+i\chi}[1+2i\sin k(L-a)\mathcal{F}e^{-2\chi a}], \tag{A1}$$

$$A_2\simeq\frac{2k}{k-i\chi}\,\frac{(k-i\chi)^2}{2\chi k}\sin k(L-a)\mathcal{F}e^{-2\chi a}, \tag{A2}$$

$$B_2\simeq\frac{k-i\chi}{k+i\chi}\left\{\frac{2k}{k-i\chi}\left[1-\frac{(k-i\chi)^2}{2\chi k}\sin k(L-a)\mathcal{F}e^{-2\chi a}\right]\right\}, \tag{A3}$$

$$A_3\simeq e^{-ikL}\mathcal{F}e^{-\chi a}, \tag{A4}$$

$$B_3\simeq\frac{k-i\chi}{k+i\chi}e^{ikL}\mathcal{F}e^{-\chi a}, \tag{A5}$$

$$A_4\simeq 0, \tag{A6}$$

$$B_4\simeq\frac{2k}{k+i\chi}\mathcal{F}e^{-\chi a}, \tag{A7}$$

$$T\simeq\frac{4i\chi k}{(k+i\chi)^2}\mathcal{F}e^{-2\chi a} \tag{A8}$$

(in all these expressions we have neglected terms of third order in $e^{\chi a}$).

[1] E.H. Hauge and J.A. Stovneng, Rev. Mod. Phys. **61**, 917 (1989); V.S. Olkhovsky and E. Recami, Phys. Rep. **214**, 339 (1992); R. Landauer and Th. Martin, Rev. Mod. Phys. **66**, 217 (1994); R.Y. Chiao and A.M. Steinberg, Prog. Opt. **37**, 345 (1997).
[2] A. Enders and G. Nimtz, J. Phys. I **2**, 1693 (1992).
[3] A.M. Steinberg, P.G. Kwiat, and R.Y. Chiao, Phys. Rev. Lett. **71**, 708 (1993); Ch. Spielmann, R. Szipocs, A. Stingl, and F. Krausz, *ibid.* **73**, 2308 (1994); G. Nimtz, A. Enders, and H. Spieker, J. Phys. I **4**, 565 (1994).
[4] Ph. Balcou and L. Dutriaux, Phys. Rev. Lett. **78**, 851 (1997); D. Mugnai, A. Ranfagni, and L. Ronchi, Phys. Lett. A **247**, 281 (1998); J.J. Carey, J. Zawadzka, D.A. Jaroszynski, and K. Wynne, Phys. Rev. Lett. **84**, 1431 (2000).
[5] M. Mojahedi, E. Schamiloglu, K. Agi, and K.J. Malloy, IEEE J. Quantum Electron. **36**, 418 (2000); M. Mojahedi, E. Schamiloglu, F. Hegeler, and K.J. Malloy, Phys. Rev. E **62**, 5758 (2000).
[6] A. Haibel and G. Nimtz, Ann. Phys. (Leipzig) **8**, 707 (2001).
[7] A. Sommerfeld, *Vorlesungen über Theoretische Physik, Band IV, Optik* (Dieterichsche Verlagsbuchhandlung, 1950); L. Brillouin, *Wave Propagation and Group Velocity* (Academic Press, New York, 1960).
[8] T.E. Hartman, J. Appl. Phys. **33**, 3427 (1962); J.R. Fletcher, J. Phys. C **18**, L55 (1985).
[9] S. Esposito, Phys. Rev. E **64**, 026609 (2001).
[10] S. Longhi, P. Laporta, M. Belmonte, and E. Recami, Phys. Rev. E **65**, 046610 (2002); A. Enders and G. Nimtz, Phys. Rev. B **47**, 9605 (1993); A. Enders and G. Nimtz, J. Phys. I **4**, 565 (1994).
[11] V.S. Olkhovsky, E. Recami, and G. Salesi, Europhys. Lett. **57**, 879 (2002).
[12] Y. Aharonov, N. Erez, and B. Reznik, Phys. Rev. A **65**, 052124 (2002).
[13] Th. Martin and R. Landauer, Phys. Rev. A **45**, 2611 (1992); R. Chiao, P. Kwiat, and A. Steinberg, Physica B **175**, 257 (1991); A. Ranfagni, D. Mugnai, P. Fabeni, and G. Pazzi, Appl. Phys. Lett. **58**, 774 (1991).
[14] C. Bracher, M. Riza, and M. Kleber, Appl. Phys. A: Mater. Sci. Process. **66**, S901 (1998).